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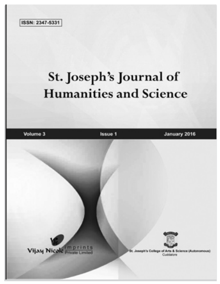

RANKING OF DODECAGONAL FUZZY NUMBERS FOR SOLVING MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM WITH SIMPLEX METHOD AND GRAPHICAL METHOD

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Abstract

In this paper, a ranking procedure based on Dodecagonal Fuzzy numbers, is applied to a Multi – Objective Linear Programming Problem (MOLPP) with fuzzy coefficients. By this ranking method any Multi Objective Fuzzy Linear Programming Problem (MOFLPP) can be converted into a crisp value problem to get an optimal solution. This ranking procedure serves as an efficient method wherein a numerical example is taken.

Keywords: Ranking, Dodecagonal Fuzzy Numbers, MOFLPP, Simplex Method, Graphical Method, α – Level Set.

INTRODUCTION

Ranking fuzzy number is used in decision – making process in an economic environment. In an organization various activities such as planning, execution, and other process takes place continuously. This requires careful observation of various parameters which are all in uncertain in nature due the competitive business environment globally. In fuzzy environment ranking fuzzy numbers is a very important decision making procedure.

The idea of fuzzy set was first proposed by Bellman and **Zadeh [1]**, as a mean of handling uncertainty that is due to imprecision rather than randomness. The concept of fuzzy linear programming (FLP) was first introduced by **Tanaka [8]** et al. **Zimmerman [2]** introduced fuzzy linear programming in fuzzy environment.

Multi-objective linear programming was introduced by **Zelenly. Lai Y.J – Hawng C.L** considered MOLPP with all parameters having a triangular possibility distribution. They used an auxiliary model and it was solved by MOLPP. **Zimmerman** applied their approach to vector maximum problem by transforming MOFLP problem to a single objective linear programming problem.

Qiu–PengGu, and **Bing–Yuan Cao** solved fuzzy linear programming based on the representation theorem and on fuzzy number ranking method. In particular, the most convenient methods are based on the concept of comparison of fuzzy numbers by the use ranking function.

PRELIMINARIES

Definition

If X is a universe of discourse and x be any particular element of X, then a fuzzy set A defined on X me written as,

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$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$

The membership function of a fuzzy set $\tilde{A}$ is denoted by$\mu_{\tilde{A}}$, i.e., $\mu_{\tilde{A}}: X \rightarrow [0, 1]$

The membership function of a fuzzy set $\tilde{A}$ has the form, $\tilde{A}: X \rightarrow [0, 1]$

Fuzzy Number

A Fuzzy set $\tilde{A}$ of the real line R with membership function

$\mu_{\tilde{A}}(x): R \rightarrow [0, 1]$ is called fuzzy number if

i) A must be normal and convex fuzzy set

ii) the support of $\tilde{A}$, must be bounded

iii) α_A must be a closed interval for every $\alpha \in [0, 1]$

Support

The support of a fuzzy set $\tilde{A}$,S ($\tilde{A}$), is the crisp set of all $x \in X$ such that,

$S(\tilde{A}) = \mu_{\tilde{A}}(x) > 0$

α – Cut set or α – Level set:

The α – Cut set of a fuzzy set $\tilde{A}$ of the set X is the following crisp set given

$\tilde{A}a = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha$

Normal Fuzzy Set

A Fuzzy set $\tilde{A}$ of a set X is said to be a normal fuzzy set iff

$\mu_{\tilde{A}}(x) = 1$ for at least one $x \in X$

DODECAGONAL FUZZY NUMBERS

A fuzzy number $\tilde{A}$ is a Dodecagonal Fuzzy Number (DoFN) denoted by

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$$

Where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}$ are real numbers and its membership function is given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ k_1\left(\frac{x - a_1}{a_2 - a_1}\right) & a_1 \leq x \leq a_2 \\ k_1 & a_2 \leq x \leq a_3 \\ k_1 + (k_2 - k_1)\left(\frac{x - a_3}{a_4 - a_3}\right) & a_2 \leq x \leq a_3 \\ k_2 & a_4 \leq x \leq a_5 \\ k_2 + (1 - k_2)\left(\frac{x - a_5}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\ 1 & a_6 \leq x \leq a_7 \\ k_2 + (1 - k_2)\left(\frac{a_8 - x}{a_8 - a_7}\right) & a_7 \leq x \leq a_8 \\ k_2 & a_8 \leq x \leq a_9 \\ k_1 + (k_2 - k_1)\left(\frac{a_{10} - x}{a_{10} - a_9}\right) & a_9 \leq x \leq a_{10} \\ k_1 & a_{10} \leq x \leq a_{11} \\ k_1\left(\frac{a_{12} - x}{a_{12} - a_{11}}\right) & a_{11} \leq x \leq a_{12} \\ 0 & a_{12} \leq x \end{cases}$$

Where $0 < k_1 < k_2 < 1$

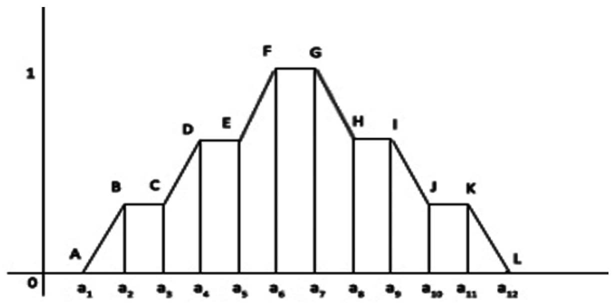


Figure: 9.1. Graphical representation of a dodecagonal fuzzy number for $x \in [0, 1]$

Arithmetic Operations on Dodecagonal Fuzzy Numbers

Let $\tilde{A}$ Do = $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ and $\tilde{B}$ Do = $(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12})$ be two Dodecagonal Fuzzy Numbers, then

i) Addition

$\tilde{A}_{Do} (+) \tilde{B}_{Do} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8, a_9+b_9, a_{10}+b_{10}, a_{11}+b_{11}, a_{12}+b_{12})$

ii) Subtraction

$\tilde{A}_{Do} (-) \tilde{B}_{Do} = (a_1- b_1, a_2- b_2, a_3- b_3, a_4- b_4, a_5- b_5, a_6- b_6, a_7 - b_7, a_8- b_8, a_9- b_9, a_{10}-b_{10}, a_{11}- b_{11}, a_{12}- b_{12})$

iii) Multiplication

$\tilde{A}_{Do} (*) \tilde{B}_{Do} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6, a_7 * b_7, a_8 * b_8, a_9 * b_9, a_{10} * b_{10}, a_{11} * b_{11}, a_{12} * b_{12})$

iv) Division

$\tilde{A}_{Do} (\div) \tilde{B}_{Do} = (a_1 \div b_1, a_2 \div b_2, a_3 \div b_3, a_4 \div b_4, a_5 \div b_5, a_6 \div b_6, a_7 \div b_7, a_8 \div b_8, a_9 \div b_9, a_{10} \div b_{10}, a_{11} \div b_{11}, a_{12} \div b_{12})$

Ranking of Dodecagonal Fuzzy Numbers

A number of approaches have been proposed for the ranking of fuzzy numbers.

In this paper for a dodecagonal fuzzy number $\tilde{A}_{Do} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ a ranking method is devised based on the following formula,

$$\mathbf{R}(\tilde{A}_{Do}) = \left(\frac{\begin{array}{l} 2(a_1 + a_6 + a_7 + a_{12}) + \\ 6(a_2 + a_3 + a_4 + a_9 + a_{10} + a_{11}) \\ +5(a_5 + a_8) \end{array}}{54} \right) \left(\frac{25}{18} \right)$$

Let $\tilde{A}_{Do} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ and $\tilde{B}_{Do} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12})$ be two Dodecagonal Fuzzy Numbers, then

$\tilde{A}_{Do} \approx \tilde{B}_{Do}$ $R(\tilde{A}_{Do}) = R(\tilde{B}_{Do})$

$\tilde{A}_{Do} \geq \tilde{B}_{Do}$ $R(\tilde{A}_{Do}) \geq R(\tilde{B}_{Do})$

$\tilde{A}_{Do} \leq \tilde{B}_{Do}$ $R(\tilde{A}_{Do}) \leq R(\tilde{B}_{Do})$

METHOD OF SOLVING MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM

This paper we discuss a multi – objective fuzzy linear programming problem in constraints conditions with fuzzy coefficients.

Maximize $Z_1 = f_1 y$

Minimize $Z_2 = f_2 y$

Subject to

$\tilde{A}_{Do} X \leq \bar{g}$, $X \geq 0$

Where $f_{ij} = (f_{i1}, f_{i2}, \ldots, f_{in})$ is an n – dimensional crisp row vector,

$\tilde{A}_{Do} = \bar{a}_{ij}$ is an m × n fuzzy matrix,

$\bar{g} = (g_1, g_2, \ldots, g_m)^T$ is an m – dimensional fuzzy line vector and

$X = (x_1, x_2, x_3 \ldots,)^T$ is an n – dimensional decision variable vector.

We now consider a bi – objective fuzzy linear programming problem with constraints having fuzzy coefficients is given by

Maximize $Z_1 = f_{11} x_1 + f_{12} x_2 + \ldots + f_{1n} x_n$

Minimize $Z_2 = f_{11} x_1 + f_{12} x_2 + \ldots + f_{1n} x_n$

Subject to

$\bar{a}_{i1} x_1 + \bar{a}_{i2} x_2 + \ldots + \bar{a}_{in} x_n \leq \bar{g}_i$

$x_1, x_2, x_3, \ldots, x_n \geq 0$, $i = 1,2,3, \ldots, m$

where fuzzy numbers are dodecagonal,

where

$$\bar{a}_{i1} = \bar{a}_{i11}, \bar{a}_{i12}, \bar{a}_{i13}, \bar{a}_{i14}, \bar{a}_{i15}, \bar{a}_{i16}, \bar{a}_{i17}, \bar{a}_{i18}, \bar{a}_{i19}, \bar{a}_{i110}, \bar{a}_{i111}, \bar{a}_{i112}$$
$$\bar{a}_{i2} = \bar{a}_{i21}, \bar{a}_{i22}, \bar{a}_{i23}, \bar{a}_{i24}, \bar{a}_{i25}, \ldots, \bar{a}_{i212}$$

....

....

$$\bar{a}_{in} = \bar{a}_{in1}, \bar{a}_{in2}, \ldots, \bar{a}_{in8}, \bar{a}_{in9}, \bar{a}_{in10}, \bar{a}_{in11}, \bar{a}_{in12}$$

$$\bar{g} = \bar{g}_{i1}, \bar{g}_{i2}, \bar{g}_{i3}, \ldots, \bar{g}_{i12}$$

By the ranking Algorithm, the above MOFLPP is transformed into a MOLPP is as follows:

Maximize $Z_1 = f_{11}x_1 + f_{12}x_2 + \ldots + f_{1n}x_n$

Minimize $Z_2 = f_{11}x_1 + f_{12}x_2 + \ldots + f_{1n}x_n$

Subject to,

$2[(a_{i11} x_1 + a_{i21}x_2 + \ldots\ldots + a_{in1}x_n) + (a_{i16} x_1 + a_{i26} x_2 + \ldots\ldots + a_{in6}x_n) +$

$(a_{i17} x_1 + a_{i27} x_2 + \ldots\ldots + a_{in7}x_n) + (a_{i112} x_1 + a_{i212} x_2 + \ldots\ldots + a_{in12}x_n)]$

$+ 6[(a_{i12} x_1 + a_{i22} x_2 + \ldots\ldots + a_{in2}x_n) + (a_{i13} x_1 + a_{i23} x_2 + \ldots\ldots + a_{in3}x_n) +$

$(a_{i14} x_1 + a_{i24} x_2 + \ldots\ldots + a_{in4}x_n) + (a_{i19} x_1 + a_{i29} x_2 + \ldots\ldots + a_{in9}x_n) +$

$(a_{i110} x_1 + a_{i210} x_2 + \ldots\ldots + a_{in10}x_n) + \ldots\ldots + (a_{in11} x_1 + a_{in11} x_2 + \ldots\ldots + a_{in11}x_n)]$

$+ 5[(a_{i15} x_1 + a_{i25} x_2 + \ldots\ldots + a_{in5}x_n) + (a_{i18} x_1 + a_{i28} x_2 + \ldots\ldots + a_{in8}x_n)]$

$\leq 2 g_{i1} + 6g_{i2} + 6g_{i3} + 6g_{i4} + 5g_{i5} + 2g_{i6} + 2g_{i7} + 5g_{i8} + 6g_{i9} + 6g_{i10} + 6g_{i11} + 2g_{i12}$

$x_1, x_2, x_3, \ldots, x_n \geq 0, i = 1, 2, 3, \ldots\ldots, m$ ——— (*)

Using ($_*$), this can be converted into a single objective problem subject to the constraints with transformed crisp number coefficients and hence solved accordingly.

Similarly, multi – objective problems with more than two objectives can also be solved using the above procedure, here in the very first stage itself the problem is transformed into a crisp problem and afterwards there will be no more fuzziness in the constraints as well as in the problem.

Simplex Method Algorithm

Step 1: Determine a starting basic feasible solution.

Step 2: Select an entering variable using the optimality condition Stop if there is no entering variable; the last solution is optimal. Else, go to Step 3.

Step 3: Select a leaving variable using the feasibility condition.

Step 4: Determine the new basic solution. Go to Step 2.

Numerical Example

Consider,

Max $Z=50 x_1+80x_2$

$\tilde{a}_{11}X_1 + \tilde{a}_{12}X_2 \leq \bar{g}_1$

$\tilde{a}_{21}X_1 + \tilde{a}_{22}X_2 \leq \bar{g}_2$

where

$\tilde{a}_{11}$ = (100, 30, 60, 110, 70, 200, 200, 105, 50, 120, 90, 65)

$\tilde{a}_{12}$ = (160, 100, 140, 50, 180, 150, 150, 170, 40, 200, 70, 90)

$\tilde{a}_{21}$ = (100, 150, 60, 90, 170, 300, 300, 190, 110, 80, 50, 200)

$\tilde{a}_{22}$ = (180, 50, 120, 70, 90, 100, 100, 110, 200, 60, 80, 40)

$\bar{g}_1$ = (800, 1400, 400, 600, 1300, 1000, 1000, 700, 900, 1600, 1200, 1100)

$\bar{g}_2$ = (500, 1000, 1100, 650, 1250, 800, 1500, 1500, 600, 1300, 400, 700)

Subject to constraints

$2(100x_1 + 160x_2) + 6(30x_1 + 100x_2) + 6(60x_1 + 140x_2) + 6(110x_1 + 50x_2) +$

$5(70x_1 + 180x_2) + 2(200x_1 + 150x_2) + 2(200x_1 + 150x_2) + 5(105x_1 + 170x_2) +$

$6(50x_1 + 40x_2) + 6(120x_1 + 200x_2) + 6(90x_1 + 70x_2) + 2(65x_1 + 90x_2)$

$\leq (800 + 1400 + 400 + 600 + 1300 + 1000 + 1000 + 700 + 900 + 1600 + 1200 + 1100)$

$2(100x_1 + 180x_2) + 6(150x_1 + 50x_2) + 6(60x_1 + 120x_2) + 6(90x_1 + 70x_2) +$

$5(170x_1 + 90x_2) + 2(300x_1 + 100x_2) + 2(300x_1 + 100x_2) + 5(190x_1 + 110x_2) +$

$6(110x_1 + 200x_2) + 6(80x_1 + 60x_2) + 6(50x_1 + 80x_2) + 2(200x_1 + 40x_2)$

$\leq (500 + 1000 + 1100 + 650 + 1250 + 800 + 1500 + 1500 + 600 + 1300 + 400 + 700)$

Max $Z = 50x_1 + 80x_2$

Subject to constraints

$4765 x_1 + 6450 x_2 \leq 12000$

$x_1 + 5320 x_2 \leq 11300$

SIMPLEX METHOD

Step 1

		C_j	50	80	0	0	
C_B	X_B	B	X_1	X_2	S_1	S_2	RATIO
0	S_1	12000	4765	6450	1	0	12000/6450←
0	S_2	11300	6760	5320	0	1	11300/5320
Z_j		0	0	0	0	0	
C_j–Z_j		-	50	80↑			

Step 2

Enter X_2 and skip S_1

		C_j	50	80	0	0	
C_B	X_B	B	X_1	X_2	S_1	S_2	RATIO
80	X_2	12000 /6450	4765 /6450	1	1 /6450	0	
0	S_2	9045000 /6450	18252200 /6450	0	-5320 /6450	1	
Z_j		148.83	59.10	80	0.01	0	
C_j- Z_j		148.83	-9.10	0	-0.01	0	

Maximize Z = 148.83 at $x_1 = 0$; $x_2 = 1.86$

Similarly,

We can calculate Minimize Z = (-Maximize Z)

Therefore,

Minimize Z = 84 at $x_1 = 1.67$; $x_2 = 0$

GRAPHICAL METHOD

Maximize $Z=50\ x_1+80x_2$

Subject to constraints

$4765\ x_1+6450\ x_2 \leq 12000$

$6760\ x_1+5320\ x_2 \leq 11300$

Solution:

Given: Max $Z=50\ x_1+80x_2$

Subject to constraints

$4765\ x_1+6450\ x_2 = 12000$ ----------→(1)

$6760\ x_1+5320\ x_2 = 11300$ ----------→(2)

Put $x_1 = 0$ in (1) ➔

$6450\ x_2 = 12000$

$X_2 = 1.86$

A (0, 1.86)

Put $x_2 = 0$ in (1) ➔

$4765\ x_1 = 12000$

$X_1 = 2.51$

B (2.51, 0)

Put $x_1 = 0$ in (2) ➔

$5320\ x_2 = 11300$

$X_2 = 2.12$

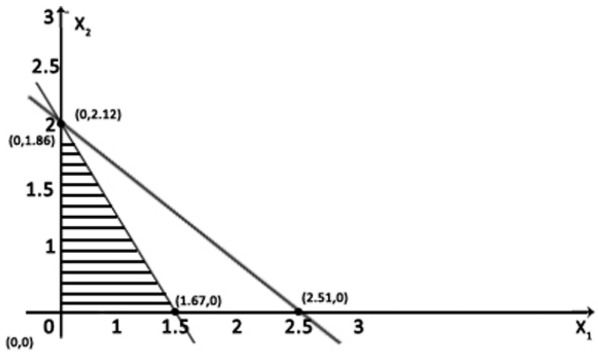
C (0, 2.12)

Put $x_2 = 0$ in (2) ➔

$6760x_1 = 11300$

$X_1 = 1.67$

D (1.67, 0)



(x_1, x_2)	Max $Z=50\ x_1+80\ x_2$
A (0,0)	Max Z = 0
B (0,1.86)	Max Z = 148.8
C (1.67,0)	Min Z = 83.55

COMPARISON OF RESULT OBTAINED BY SIMPLEX METHOD AND GRAPHICAL METHOD

Graphical Method	Simplex Method
Maximize Z=148.8 at x_1=0 and x_2=1.86	Maximize Z=148.83 at x_1=0 and x_2=1.86
Minimize Z=83.55 at X_1=1.67 and x_2=0	Minimize Z=84 at X_1=1.7 and x_2=0

RESULT

From the above table, we have obtained the correct value from both Simplex Method and Graphical Method. By comparing the two methods Simplex Method is the best one in which the results are more accurate.

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